Institute of Research and Publication Indonesia
IJIRSE: Indonesian Journal of Informatic Research and Software Engineering
Journal Homepage: https://journal.irpi.or.id/index.php/ijirse
Vol. 5. No.1. Maret 2025, pp: 79 - 87
E-ISSN: 2775-5754 | P-ISSN: 2797-2712

Algorithm Design and Programming the Luenberger Observer for level estimation in a Storage Tank System

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Makalah: Diterima 30 November 2024; Diperbaiki 20 Februari 2025; Disetujui 25 Maret 2025
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Abstract

This study presents the design and implementation of a Luenberger Observer algorithm for state estimation in a liquid storage tank system. The methodology includes system parameter identification, data preprocessing, observer gain calculation using pole placement, and simulation in MATLAB and Simulink. To reflect real-world conditions, synthetic disturbances were added and the input signal was normalized to improve numerical stability. Quantitative evaluation was conducted by comparing the system output with the observer's estimated output. Simulation results demonstrate that the observer effectively tracks the system dynamics, yielding a root mean square error (RMSE) of 7×10^{-5} m and a near-zero steady-state error. The observer's robustness was also tested systematically through increasing levels of synthetic measurement noise, showing stable and accurate performance even under 6% noise conditions. These findings confirm that the proposed algorithm provides reliable and responsive state estimation, with strong potential for practical application in control systems for dynamic fluid environments.

Keyword: algorithm, programming, Luenberger observer, estimation, storage tank system

1. INTRODUCTION

Accurate estimation of a system's state is crucial because real-world systems are often affected by noise, disturbances, and incomplete measurements [1]. Estimation enables us to understand and monitor complex dynamic systems in real time, even when direct measurements are noisy or partially unavailable. It improves control and decision-making by providing reliable information about the system's current condition, optimizes performance and safety in industries such as robotics, aerospace, automotive, and smart grids, and enables predictive maintenance and fault detection by estimating hidden or unmeasurable variables [2-4]. Furthermore, it supports autonomous systems that rely heavily on estimation algorithms to navigate and interact with their environment. In short, estimation bridges the gap between imperfect measurements and the true state of a system, making it fundamental in modern technology and engineering [5, 6]. Therefore, accurate estimation must take into account robustness, which refers to how well the estimator performs even under noisy measurements, uncertainties, and disturbances. Robust algorithms help maintain stability and accuracy even when the environment or system parameters change unexpectedly [7]. They also prevent failures or degraded performance in real-world conditions where perfect models and data are rarely available. Since many real-world systems are nonlinear and subject to unpredictable disturbances. Robust algorithm design (e.g., Kalman filters, particle filters, observers) allows the estimator to handle these complexities effectively.

The LO is an effective state estimation technique for linear time-invariant (LTI) systems, allowing the reconstruction of unmeasured internal states using available input and output data. It operates by correcting estimation errors through feedback from the output error, ensuring convergence of the estimated states to the true values . To guarantee stability and achieve desired dynamic performance, the observer gain is determined using the pole placement method, which assigns the observer error dynamics' poles to specific locations in the complex plane which is typically faster than the system poles, to ensure rapid and stable convergence [8-10]. The proposed algorithm offers several advantages, including model-based precision, fast convergence, computational efficiency, and ease of integration with control systems. It is well-suited for real-time applications and enhances system observability, making it valuable in various engineering domains where accurate and reliable state estimation is essential. Despite its proven effectiveness in theory, the application of the LO in storage tank systems, especially under realistic disturbances and noisy measurement conditions, has not been extensively investigated. Many prior studies focus on ideal or noise-free environments, limiting the practical relevance of their findings. Additionally, few works integrate comprehensive signal preprocessing techniques, such as normalization and synthetic noise modeling, in conjunction with observer implementation to reflect real-world sensor behaviors. These gaps limit the understanding of how LO perform under industrial conditions where robustness is critical.

Building on the effectiveness of the LO in providing accurate and reliable state estimation, its practical implementation becomes a crucial step in validating performance under real-world conditions. To achieve this, MATLAB and Simulink are used as the primary tools due to their powerful capabilities in modeling dynamic systems and developing control algorithms [11, 12]. MATLAB facilitates the numerical computation and coding of the observer, including the design of the gain matrix using the pole placement method, while Simulink offers a graphical interface for simulating the entire system in a modular and intuitive manner [13]. A key aspect of this implementation is signal preprocessing, which includes steps such as noise modeling, to emulate realistic sensor disturbances, and normalization, which ensures that signal magnitudes are consistent and numerically stable. These preprocessing steps are essential to accurately assess the observer's performance in a noisy and uncertain environment. The observer is then integrated into the system model within Simulink to perform real-time state estimation, allowing for closed-loop simulations that test responsiveness, robustness, and convergence under various operating scenarios. This implementation not only validates the algorithm's theoretical design but also demonstrates its applicability in real-time control systems.

To demonstrate the practical relevance of the proposed observer algorithm, a case study involving a storage tank system is implemented. Storage tanks are a crucial component in process engineering and are widely used at both small and large scales across various industries [14]. Their use is especially prevalent in chemical sectors such as oil and gas, petrochemicals, and polymers. These tanks serve as essential containers for storing various types of liquids or chemicals, depending on industrial requirements. Monitoring and controlling the liquid level or flow within these tanks is vital for ensuring process safety, efficiency, and product quality [15, 16]. However, due to sensor limitations and environmental disturbances, not all internal states can be directly measured. This makes state estimation techniques like the LO highly valuable, as they provide accurate and real-time estimation of unmeasurable variables, such as internal fluid dynamics, based on available input and output data. Nevertheless, prior research on observer-based estimation in storage tanks rarely considers comprehensive noise scenarios and signal conditioning practices [17, 18]. Addressing this deficiency, the present study applies the LO in a simulation framework that explicitly includes noise modeling, disturbance injection, and data normalization, thereby aligning the experimental setup with industrial conditions. Integrating the observer into the simulation of a storage tank system in MATLAB/Simulink allows for a realistic assessment of its performance under operational conditions, reinforcing its potential for deployment in real industrial processes.

Based on this context, the research objectives are defined as follows: to design and program the LO algorithm for fluid level estimation in a storage tank system; to validate its performance through simulation under both ideal and noisy measurement conditions; and to contribute to the practical implementation of observer-based estimation in fluid storage applications. The scope of the study includes the development of the observer using MATLAB/Simulink, the incorporation of signal preprocessing techniques, and the performance evaluation through closed-loop simulation. Finally, the structure of this paper is organized as follows: the Method section outlines the system modeling, observer design, and simulation setup (algorithm implementation); the Results and Discussion section presents the performance analysis under various test conditions; and the final part presented some Conclusion.

2. RESEARCH MATERIALS AND METHODS

The methodology of this study involves several main stages: data collection for system modeling, data preprocessing (including noise modeling and signal normalization), observer gain design using the pole placement method, system simulation, and performance analysis. The overall research process is illustrated in Figure 1.

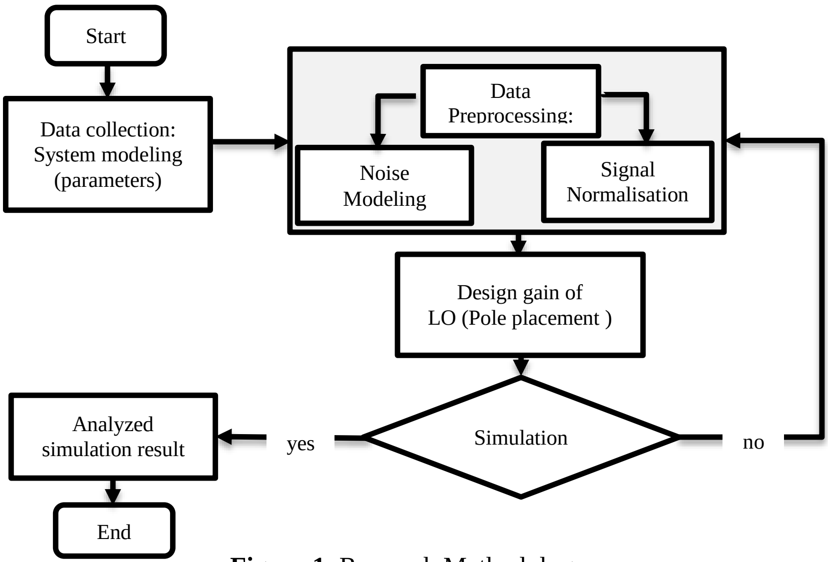


Figure 1. Research Methodology

Figure 1 shows the sequential process of this study, starting with collecting key system parameters such as tank dimensions, flow rates, initial fluid levels, and sensor data for accurate modeling. Data preprocessing follows, involving noise modeling with synthetic disturbances and signal normalization to improve data quality and stability. The LO gain is then designed using the pole placement method to ensure desired dynamic response and stability. Simulations are performed in MATLAB and Simulink to enable real-time state estimation. Performance analysis evaluates the observer's sensitivity, robustness to noise, estimation accuracy, convergence speed, and noise tolerance. Finally, the results confirm the observer's effectiveness in accurately estimating internal states under both ideal and noisy conditions.

2.1 System Modeling

The mathematical model of the storage tank system is derived based on the volume balance law, which relates the change in liquid volume to the inflow and outflow rates, as shown in Figure 2.

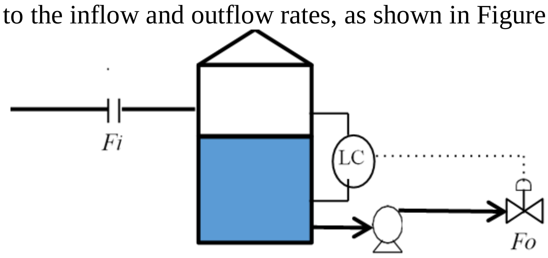


Figure 2. Storage tank system

In this system, liquid enters the tank via a Flow Indicator (Fi), while a Level Controller (LC) maintains the liquid level within safe limits to prevent overflow or underflow. The liquid exits through a Flow Outlet (Fo), regulated by a control valve. Based on the volume balance principle, the rate of change in tank volume is governed by the difference between inflow and outflow, and is described by the following differential equation:

$$\frac{dV(t)}{dt} = F_i(t) - F_o(t) \tag{1}$$

where $V(t) = Ah(t)$, hence

$$\frac{dV(t)}{dt} = A\frac{dh(t)}{dt} \tag{2}$$

by substituting (2) into (1) the result is obtained as follows :

$$\frac{dh(t)}{dt} = \frac{1}{A}F_i(t) - \frac{1}{A}F_o(t) \tag{3}$$

The outflow rate is given by $c\sqrt{h(t)}$, therefore

$$\frac{dh(t)}{dt} = \frac{1}{A}F_i(t) - \frac{c\sqrt{h}(t)}{A} \tag{4}$$

The mathematical model of the storage tank system is nonlinear, therefore linearization is required to simplify the analysis. Considering non-linear system as follows:

$$\dot{x}(t) = f(x,u,t)$$
$$y(t) = h(x,u,t) \tag{5}$$

using the Taylor Series Expansion as shown below [19]:

$$f(x,u) \approx f(x_0,u_0) + \frac{\partial f}{\partial x}\Big|_{(x_0,u_0)}(x - x_0) + \frac{\partial f}{\partial u}\Big|_{(x_0,u_0)}(u - u_0) \tag{6}$$

Because $f(x_0,u_0) = 0$ at the equilibrium point (steady state), the linear equation becomes:

$$\dot{x}(t) = A\big(x(t) - x_0(t)\big) + B\big(u(t) - u_0(t)\big)$$
$$y(t) = C\big(x(t) - x_0(t)\big) + D\big(u(t) - u_0(t)\big) \tag{7}$$

with $A = \frac{\partial f}{\partial x}\Big|_{(x_0,u_0)}$, $B = \frac{\partial f}{\partial u}\Big|_{(x_0,u_0)}$, $C = \frac{\partial h}{\partial x}\Big|_{(x_0,u_0)}$, and $D = \frac{\partial h}{\partial u}\Big|_{(x_0,u_0)}$. The linearisation of storage tank system model is shown as follows:

$$\dot{x}(t) = \left[-\frac{c}{2A\sqrt{h(t)}}\right]x(t) + \left[\frac{1}{A}\right]u(t)$$
$$y(t) = [1]x(t) + [1]u(t) \tag{8}$$

with matrices A, B, C, and D defined as

$$\Phi = \left[-\frac{c}{2A\sqrt{h(t)}}\right]; B = \left[\frac{1}{A}\right]; C = [1]; D = [0]. \tag{9}$$

The matrix A, which was previously used to represent the values of the storage tank system, has been renamed by Φ. This change was made to avoid confusion, since the symbol A is already used to denote the tank's cross-sectional area. The parameters of the storage tank system are presented in Table 1.

Table 1. Parameters of storage tank system

Parameters	Symbol and unit
Diameter	d (1m)
Level	h (0.75m)
Cross-sectional area	A ($\pi 4d^2$ m^2)
Flow input	F_i (0.4 m/s)
Contant	c (0.02 SI)

Therefore, the linear system of the storage tank system becomes:

$$\dot{x}(t) = \Phi x(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{10}$$

where $x(t)$ represents the state variable, $y(t)$ is the measurement variable, and $u(t)$ is input system. $\Phi, B, C,$ and D are the known matrices. Vector *v* represents Gaussian measurement noise. This formulation serves as the foundation for designing the LO for the storage tank system.

2.2 Design of Luenberger Observer (LO)

A LO is a method in control systems used to estimate the internal states of a dynamic system that cannot be measured directly. The LO uses a linear system model. A continuous-time linear system in state-space form is written as:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + v(t) \tag{11}$$

where $x(t) \in \mathbf{R}^1$ is the continuous state vector, $y(t) \in \mathbf{R}^1$ is a vector of discrete measurement, and $u \in \mathbf{R}^1$ continuous input vector, $A \in \mathbf{R}^{3\times3}, B \in \mathbf{R}^{3\times2}$, and $C \in \mathbf{R}^{2\times3}$ are known matrix. Vector *v* represents Gaussian measurement noise. The algorithm of LO can be written as follows:

$$\dot{\hat{x}}(t) = Ax(t) + Bu(t) + K(t)(y(t) - \hat{y}(t)) \tag{12}$$

where $\hat{x}(t)$ is the estimate of $x(t)$, $K(t)$ the observer gain matrix, and $y(t) - C\hat{x}(t)$ is the residual or measurement error between the actual output and the estimated output. The derivative of the estimation error is $\dot{e}(t) = (A - KC)e(t)$. If all eigenvalues of the matrix $A - KC$ lie in the left half of the s-plane (i.e., have negative real parts), then $e(t) \to 0$ as $t \to \infty$, and the observer is considered stable. A necessary condition for designing the observer is that the system must satisfy the observability condition, meaning the pair *(C,A)* must allow reconstruction of the entire system state from its output.

2.3 Algorithm design

In this section, the algorithm is designed. Before proceeding to the observer, simulating the system alone is important to understand its behavior. Therefore, an algorithm is also needed for the simulation. Based on (8) , the Simulink block is designed as shown in Figure 3.

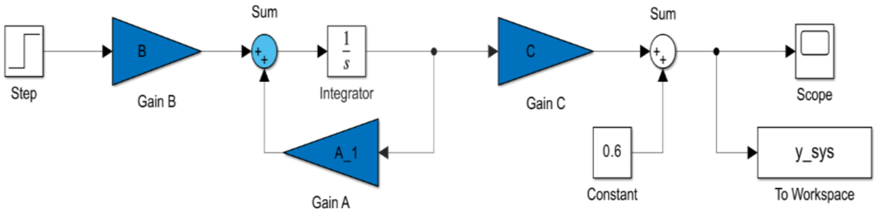


Figure 3. Design block Simulink of the storage tank system

By combining MATLAB code and Simulink callbacks, the algorithm for the storage tank system is presented in Algorithm 1

Algorithm 1. Simulation of storage tank system

Input: System parameters from Table 1
Output: Level of storage tank system (h); datasystem
Start
Initialization:
Clear command window, workspace, and figures:
clc; clear all; close all;
Set time parameters:
t_start = 0
t_sample = 1
t_end = 360
Set initial condition:
init = 0
Define system matrices:
Φ, B, C, D (from (9))
Load Simulink model:
model_name = "storagetanksystem"
load_system(model_name)
Run simulation:
sim(model_name)
Store simulation output:
datasystem = simout
Plot results:
plot(datasystem)
End

The Luenberger Observer (LO) algorithm for liquid level estimation is presented in Algorithm 2. Its Simulink implementation, based on equations (8) and (11), is shown in Figure 4, illustrating how the observer integrates model dynamics and measurements to estimate system states. Estimation errors are minimized by adjusting internal states based on output discrepancies. Algorithm 2 summarizes the computation of the observer gain and the state update process.

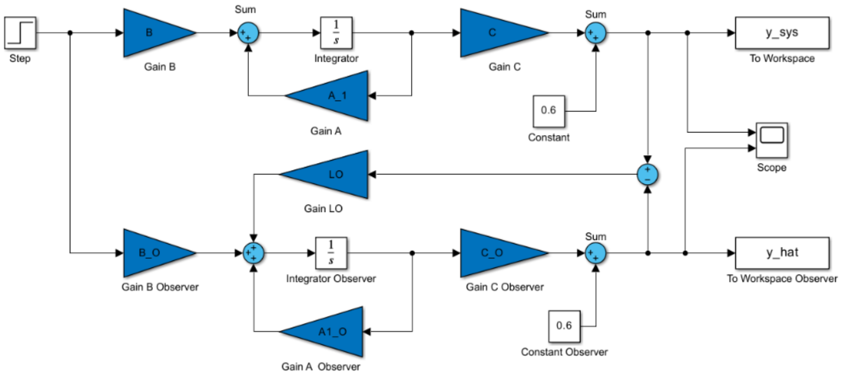


Figure 4. Design block Simulink of the storage tank system and the LO

Algorithm 2. LO for estimating level in storage tank system

```
Input: System parameters from Table 1
Output: Estimated state and output variables
Start
Initialization:
   Clear command window, workspace, and figures:
      clc; clear all; close all;
    Set time parameters:
      t_start = 0
      t_sample = 1
      t_end = 360
    Set initial system condition:
       init = 0
    Define system matrices:
      Φ, B, C, D (from (9))
    Set initial observer condition:
       init_O = 0
    Define observer matrices:
       A1_O = Φ
       B_O = B
       C_O = C
       D_O = D
     Define pole system:
        Sys=ss (A1_O, B_O, C_O, D_O)
        Pol=pole (Sys)
     Define observer gain matrix (using pole placement):
         K=place(A',C',Pol)
     Load Simulink model:
         model_name = "storagetank_Observer"
         load_system(model_name)
     Run simulation:
         sim(model_name)
      Plot results:
         plot(simout)
End
```

Using this algorithm, the simulation outputs will be examined to evaluate the performance of the LO in estimating the liquid level of the storage tank. Key metrics such as estimation accuracy, convergence speed, and robustness against measurement noise will be analyzed. Additionally, graphical representations will be used to compare the estimated states with the actual system states, providing insight into the observer's effectiveness. To assess robustness against measurement noise, appropriate disturbances must be introduced. In this study, white Gaussian noise is used, characterized by the distribution $v(t) \sim \mathrm{N}\ (0, R)$. The algorithm for generating and incorporating measurement noise is shown in Algorithm 3.

Algorithm 3. Measurement Noise Injection for Storage Tank System

```
Input: Measurement data of the storage tank system
Output: Noisy measurement data with white Gaussian noise
Start
Initialization:
   Clear command window, workspace, and figures:
      clc; clear all; close all;
   Load measurement data:
   load('measurement_data.mat')    %Assume data is stored in variable y_meas
Compute 6% of the maximum measurement value:
   y_max_6 = 0.06 * max(y_meas)   %Name this as y
Generate a random signal with the same length as y_meas:
   sigma = randn(size(y_meas));
Apply standard normalization (zero mean, unit variance):
   sigma_b = (sigma - mean(sigma)) / std(sigma);
Scale the normalized signal by 6% of the maximum measurement:
   v = y_max_6 * sigma_b;
Add the scaled noise to the original measurement:
   y_noisy = y_meas + v;
End
```

with the algorithm implemented, the next step involves analyzing the Results and Discussion section, focusing on the performance evaluation of the LO based on the simulation outputs.

3. RESULTS AND DISCUSSION

3.1 Open-loop response

The open-loop test of the double-tank system was conducted using Simulink MATLAB with Algorithm 1. The simulation started at 0 seconds and ended at 360 seconds, with a sampling time of 1 second. In this test, the system operated without feedback from the output to the input, allowing the system's behavior to be observed in an open-loop condition without any automatic correction for disturbances or output changes. The results of this test are shown in Figure 3.

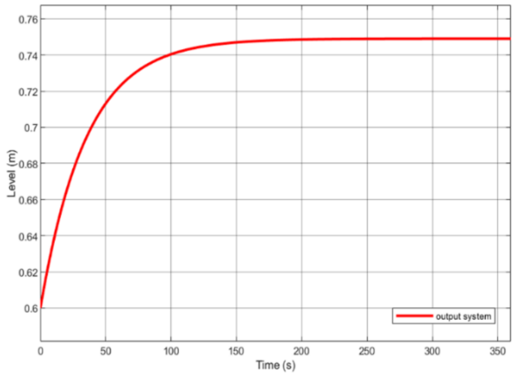
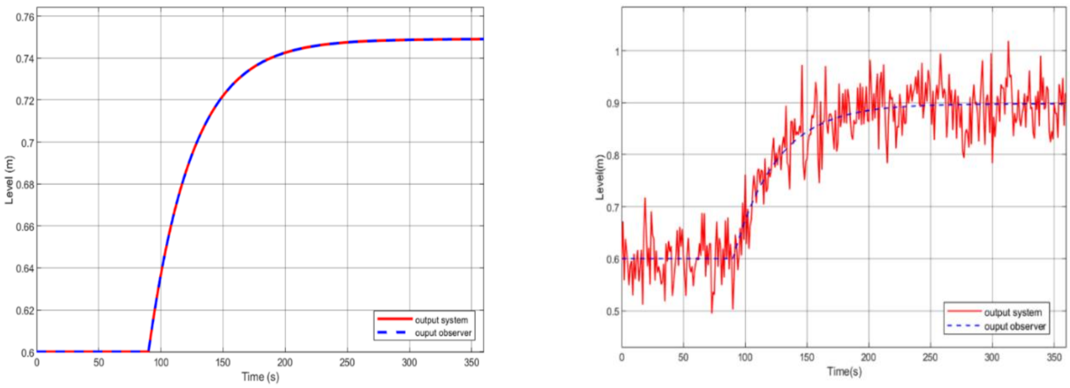


Figure 3. Open loop response for storage tank system

As shown in Figure 3, the system responds quickly to input before gradually stabilizing around a fluid level of 0.75 meters. This confirms the system's stable behavior, which provides a suitable foundation for implementing the LO algorithm to estimate the fluid level.

3.2 State estimation performance

The performance of the observer is evaluated using two main indicators, sensitivity and robustness. Sensitivity analysis is conducted to assess how responsive the observer is to variations in the output. In the simulation, two different output profiles are applied during two-time intervals: 0–90 seconds and 90–360 seconds. The results of this simulation are presented in Figure 4a. For the robustness analysis, measurement noise is introduced into the simulation. The noise is modeled as $v(t) \sim \mathrm{N}\left(0, R\right)$, with a noise covariance value of *R=0.002*. The purpose of adding this noise is to evaluate the observer's resilience to disturbances that commonly occur in real-world conditions. The state estimation results under these conditions are presented in Figure 4b.



(a) (b)

Figure 4. (a) Output results of the storage tank under input changes, (b) Output result of the storage tank under measurement noise

From Figure 4(a), it can be observed that an output variation was introduced at the 90-second mark, resulting in a rise in the fluid level before eventually stabilizing around 0.75 meters. The observer's ability to track this change demonstrates good sensitivity to output variations and the capability to adjust its estimation quickly and accurately. The estimation error, calculated using the Root Mean Square Error (RMSE), is as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_{est,i} - y_{true,i})} = 7\times10^{-5}\, m \tag{13}$$

This small error value indicates that the estimation error is minimal, demonstrating that the LO performs well in estimating the system state. Meanwhile in Figure 4(b), illustrates that, despite significant fluctuations in the system output caused by 6% measurement noise, the LO implemented using the proposed algorithm maintains accurate state estimations. This outcome confirms the algorithm's effectiveness in designing an observer that is both sensitive to system dynamics and robust against measurement disturbances. The ability of the observer to sustain reliable performance under noisy conditions demonstrates the success of the algorithm and programming approach used in this study.

4. CONCLUSION

This study successfully demonstrates the design and implementation of a Luenberger Observer (LO) for state estimation in a storage tank system. The methodology included system modeling, signal preprocessing, observer gain design, and simulation. The observer performed reliably under both ideal and noisy conditions, accurately tracking state variations and maintaining stability. Quantitative evaluation yielded a root mean square error (RMSE) of 7×10^{-5} meters, confirming high estimation accuracy. The algorithm exhibited fast convergence, low computational demand, and robustness against measurement noise and parameter variations. These characteristics support its suitability for real-time control applications, highlighting the LO as an effective and integrable solution for fluid monitoring in dynamic industrial systems.

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